Optimized Planning of Chargers for Electric Vehicles in Distribution Grids Including PV Self-Consumption and Cooperative Vehicle Owners

Abstract—**This paper presents a mathematical model to site and size the charging infrastructure for electric vehicles (EVs) in a distribution grid to minimize the required capital investments and maximize self-consumption of local PV generation jointly. The formulation accounts for the operational constraints of the distribution grid (nodal voltages, line currents, and transformers' ratings) and the recharging times of the EVs. It explicitly models the EV owners' flexibility in plugging and unplugging their vehicles to and from a charger to enable optimal utilization of the charging infrastructure and improve self-consumption (cooperative EV owners). The problem is formulated as a mixed-integer linear program (MILP), where nonlinear grid constraints are approximated with linearized grid models.**

Index Terms—**Electric vehicles; Charging stations; Siting; PV self-consumption.**

I. Introduction

The increasing population of electric vehicles (EVs) motivated the necessity of developing an extended charging infrastructure. According to [1], [2] in France, 2 billion euros will be necessary to deploy 7 million public and private chargers by 2030. Also, it is estimated in [3] that, during 2019-2025, more than 2 billion dollars will be necessary to improve the public and residential charging infrastructure across major metropolitan areas of the United States. A large number of charging stations and the simultaneous charging of many EVs might result in increased power flows, violating the operational constraints of distribution grids (voltage levels, line ampacities, and substation transformers rating). Thus, besides the investment associated with developing the charging infrastructures, additional investments might be required to upgrade and reinforce the grid infrastructures, especially distribution grids. This motivates planning the EV charging infrastructure (in terms of their locations in numbers) while cognizant of the constraints of distribution grids and driving demand of the EV owners.

A solution to reduce grid congestions and, at the same time, reduce grid losses and improve the carbon footprint of the recharging process is to charge EVs by using electricity produced by local photovoltaic (PV) generation. This paradigm, known as PV self-consumption, has been widely advocated in the literature as a way to integrate more PV into existing distribution grids [4], [5], delaying expensive grid reinforcement.

In this paper, we tackle planning the charging infrastructure for EVs, namely establishing the location and number of chargers in a distribution grid to satisfy the recharging demand of the EV owners. The objective of the study is to verify whether optimizing the EV charging infrastructure under different criteria leads to significantly different infrastructure requirements. These different criteria are: minimizing the total investment costs (i.e., chargers are planned to minimize the capital investment), optimizing for PV self-consumption (i.e., joint optimization of capital investments and facilitating PV self-consumption), reduction of the cost of recharge for end users (i.e., joint optimization of capital and operating costs under time-of-use electricity tariffs), different levels of flexibility of the EV owners in plugging and unplugging their EVs into and from chargers, and duration of the parking intervals.

The problem is formulated as a mixed-integer linear program (MILP) whose formulation is adapted to capture the different conditions described above.

The problem of planning the charging infrastructure for EVs has been investigated extensively. The works in [6] and [7] considered distribution grid and traffic flow models to identify appropriate nodes where to allocate EV charging stations with a genetic algorithm to tackle nonconvexities of AC load flow models, however without considering PV self-consumption and explicit models of the EV owners' flexibility. The work in [8] proposed a data-driven approach to identify the driving demand and locations of the chargers without however considering grid constraints, PV self-consumption and EV owners' flexibility. Works in [9], [10] proposed a two-stage optimization framework to co-optimize the charging infrastructure of EVs in combination with the operations of the power and gas networks; however, the work does not specifically address drivers' flexibility and PV self-consumption. The works in [11], [12] tackled planning of the charging infrastructure without, however, including grid constraints, PV self-consumption and EV owners' flexibility. The work in [13] proposes joint planning of EV charging stations and distribution capacity expansion without, however, modeling EV owners' flexibility and PV self-consumption. In [14] authors modeled the bounded rational charging behavior of EV drivers and applied the behavior model in solving the planning problem for EV chargers; this work did not investigate the impact of PV self-consumption. A multi-objective planning model for EV chargers is developed in [15]: even if this work considers renewable generation with wind power, it does not explore the impact of PV self-consumption or drivers' flexibility. A multi-objective planning model for the layout of an electric vehicle charging station is proposed in [16], without considering the distribution grid's operational constraints and PV self-consumption. Recently, the work in [17] presented a stochastic planning model for EV chargers, including PV generation; however, this work does not investigate the impact of PV self-consumption modeling on the planning.

In view of the existing literature, the main contribution of this paper is a quantitative investigation of the sensitivity of the EV charging infrastructure to different optimization objectives, including PV self-consumption. The question we want to answer is whether changing the planning objective leads to a substantially different charging infrastructure or whether similar configurations of charging infrastructures can be suitable to accommodate different objectives. This question is of interest for urban planners or policymakers to identify, for example, charging infrastructure that could become quickly obsolete if planning objectives change over time. From a methodological perspective, we introduce an extensible mathematical optimization model that can be used to perform this analysis.

The rest of this paper is organized as follows. Section II presents the problem statement and problem formulation. Section III describes the case study, Section IV presents and discusses the results, and finally Section V concludes the paper.

II. Planning the EV charging infrastructure

The problem formulation aims to identify the location and numbers of EV chargers in a distribution grid to satisfy the EV owners' driving needs while obeying the distribution grid's constraints. This formulation is then cast to achieve different objectives:

- optimization of the total capital investments for the whole charging infrastructure;
- optimization of the total capital investments in combination with fostering a charging infrastructure conducive to promoting PV self-consumption;
- joint optimization of the capital and operational costs (i.e., recharging costs for the EV owners considering time-of-use electricity tariffs);

The adopted formulation explicitly models the flexibility of the EV owners in plugging and unplugging their vehicles in and from chargers. The reason for this is that more flexible owners increase the utilization factor of the charging infrastructure, thus requiring planning fewer chargers for the same charging demand.

The formulation of this paper modifies and extends the planning method originally described in [18]. The methods from [18] are summarized in this section for the sake of clarity; the extension to PV self-consumption and joint optimization of capital and operational costs, the main methodological contributions of this work, is then discussed in sections (II-G) and (II-H). The problem is formulated as a constrained economic optimization program, as explained in this section.

A. Indexes and notation

Let $v = 1, \dots, V$ denote the index of the EVs, $t = 1, \dots, T$ the index of the time interval, and $n = 1, \dots, N$ the index of the node of the distribution grid, where V, T, and N are the total number of vehicles, time intervals and grid nodes, respectively. With abuse of notation and more compact expressions, we denote with subscripts nvt quantities for node n, vehicle v, and at time interval t, and not the product among these indexes; similarly for other subscripts.

B. Meeting the driving demand

The need to satisfy the driving demand is modeled by requiring all vehicles' state of charge (SOC) to be within a given range at all time. Say $\mathrm{SOC}_v(t)$ is the state-of-charge of vehicle v at time interval t, this requirement reads as:

$$\overline{\mathrm{SOC}} \leq \mathrm{SOC}_v(t) \leq \underline{\mathrm{SOC}}, \qquad \text{for all } t \text{ and } v \tag{1a}$$

where parameters $\overline{\mathrm{SOC}}$ and $\underline{\mathrm{SOC}}$ denote feasible bounds of the state-of-charge (e.g., 10% and 90% respectively).

The SOC's evolution in time is modeled as a function of discharging power (dictated by the driving demand) and the recharging power (a variable of the optimization problem). Formally, this is as:

$$\mathrm{SOC}_v(t) = \mathrm{SOC}_v(0) + \frac{T_s}{E_v} \sum_{\tau=0}^{t-1} \left(\eta \cdot p_{v\tau}^{\text{EV+}} - p_{v\tau}^{\text{EV}-} \right), \tag{1b}$$

where $\mathrm{SOC}_v(0)$ is the initial state-of-charge, T_s is the integration time in hours, E_v is the energy capacity in kWh of the battery of vehicle v, η is the charger efficiency, $p_{v\tau}^{\text{EV+}}$ is the recharging power, and $p_{v\tau}^{\text{EV-}}$ is the discharging power.

Both $p_{v\tau}^{\text{EV+}}$ and $p_{v\tau}^{\text{EV-}}$ are non-negative and mutually exclusive by construction (because an EV cannot be driven and recharged at the same time), as it will be explained later. The charging power $p_{v\tau}^{\text{EV+}}$ is a variable of the optimization problem and modeled as it will be described in (II-C).

The discharging power $p_{v\tau}^{\text{EV-}}$ is an input of the problem and thus assumed given. In this paper, it is estimated from historical measurements of the EV state of charge: for this reason, in (1b), it is not weighted by the efficiency. Input data sets are discussed in the case study section.

The model in (1b) assumes constant battery voltage and efficiency: it is commonly adopted in the literature because it is linear in the recharging power (e.g. [19]). These assumptions trade-off accuracy for increased model tractability and are considered acceptable in a planning problem with sparse temporal resolution (e.g., tens of minutes), as in this paper.

C. Modeling the charging power and constraints

This section explains how the charging power of the EVs, i.e. $p_{vt}^{\text{EV+}}$ in (1b), is computed and vehicles' recharging constraints.

We introduce the binary variable s_{vt}^{charge}, that denotes whether a vehicle v is charging at time interval t (if 1) or not (if 0). It is worth remarking that s_{vt}^{charge} for all v and t are variables of the optimization problem, meaning that their values are an output of the problem and computed by the optimization solver to meet problem's constraints and minimizing the cost function.

Say $\bar{S}$ is a charger's kVA rating and $\cos\phi$ its power factor. Assuming the charger operates on-off, the EV recharging power $p_{vt}^{\text{EV+}}$ is modeled as a function of s_{vt}^{charge} as:

$$p_{vt}^{\text{EV+}} = s_{vt}^{\text{charge}} \cdot \bar{S} \cdot \cos\phi, \qquad \forall t \text{ and } v. \tag{2}$$

As a contribution of this paper compared to the former formulation in [18], we model the capability of a charger to modulate its output power. Assuming the charger works at a constant power factor (regardless of its power output), the previous model can be modified as:

$$0 \leq p_{vt}^{\text{EV+}} \leq s_{vt}^{\text{charge}} \cdot \bar{S} \cdot \cos\phi \tag{3}$$

We introduce the binary variable s_{vt}^{plugged} to indicate whether a vehicle v is plugged into a charger at time interval t (if 1), or it is not (if 0). s_{vt}^{plugged} is also a variable of the optimization problem and is used to identify which chargers are occupied at a given time interval t.

Some constraints apply to variables s_{vt}^{charge} and s_{vt}^{plugged} to ensure a logically meaningful model, as now discussed.

First, because an EV can charge only when it is plugged into a charger, s_{vt}^{charge} can be 1 only if s_{vt}^{plugged} is 1 too. Formally, this reads as:

$$s_{vt}^{\text{charge}} \leq s_{vt}^{\text{plugged}} \qquad \forall t \text{ and } v \tag{4}$$

Second, a vehicle can be plugged into a charger only if it is parked (*plugged-in-only-if-parked* constraint). Assuming that the information on whether a vehicle is parked is available in the following input information:

$$p_{nvt} = \begin{cases} 1, & \text{if EV } v \text{ is parked at node } n \text{ at time } t \\ 0, & \text{otherwise,} \end{cases} \tag{5}$$

the *plugged-in-only-if-parked* is formalized as:

$$s_{vt}^{\text{plugged}} \leq \sum_{n=1}^{N} p_{nvt} \qquad \forall t \text{ and } v. \tag{6}$$

Input quantities p_{nvt} in (5) are assumed known from vehicles utilization data or statistics. It is worth highlighting that because a vehicle can be parked at one node only at a given time, a property of the input information p_{nvt} in (5) is that

$$\sum_{n=1}^{N} p_{nvt} \leq 1 \qquad \text{for all } t \text{ and } v. \tag{7}$$

D. Need for chargers and capital investment for the charging infrastructure

This section explains how the number and locations of the chargers in the distribution grid is determined based on the problem variables.

In words, the number of required chargers at a given grid node is identified by evaluating the maximum number of vehicles at that node which are simultaneously plugged into a charger. We denote the number of chargers required at node n by S_n^{chargers}. Its formal definition is now introduced and then explained:

$$S_n^{\text{chargers}} = \max_t \left\{ \sum_{v=1}^{V} \left(p_{nvt} \cdot s_{vt}^{\text{plugged}} \right) \right\}, \; n = 1, \ldots, N. \tag{8}$$

Eq. (8) first computes the product between p_{nvt} and s_{vt}^{plugged} for a fixed node, time interval, and vehicle. This product enables to link the vehicle (and whether it is keeping a charger busy because plugged-in) with its position in the grid. Summing over all vehicles for a fixed node and time tells the total number of vehicles plugged into a charger at the grid location n. Finally, the maximum operator computes the largest number of vehicles plugged-in at node n over time, which is the number of chargers that is necessary to have to satisfy the demand for chargers at that node.

Based on the required numbers of chargers computed in (8), one can estimate the capital cost of the charging infrastructure as

$$J^{\text{chargers}} = \sum_{n=1}^{N} \gamma \cdot S_n^{\text{chargers}} \tag{9}$$

where γ is the unitary cost of a charger.

Naturally, Eq. (9) is the function to minimize to achieve minimum costs of the charging infrastructure. It will be used later to formulate the cost function of the optimization problem.

E. Modeling plugging and unplugging behavior of EV owners

The plugged-in state of a vehicle does not depend only on its parking state, as discussed above for (6), but also on whether its owner is available to plug it into a charger (or unplug it from a charger so as to make available the charger to another EV owner). Because EV owners' behavior ultimately affects the utilization of the chargers, it also affects the number of chargers to install, thus it is important to model this element. In order to do so, additional (linear) constraints are added for the variables s_{vt}^{plugged}.

We model two scenarios of EV owners' flexibility: *forgetful EV owners* and *cooperative EV owners*. To explain these scenarios, we specifically refer to the case study considered in this paper, which considers a trip-around home/work commute, where EV owners drive their vehicles from an origin node to a destination node in the morning, and then back to the origin node in the evening. The scenarios are here explained here verbosely. The notation and formulation are explained in Appendix A.

Forgetful EV owners In both the morning and afternoon commutes, owners plug their vehicles at the arrival time and unplug them at the departing time. In these intervals, the chargers are busy for the whole duration of the parking stay, regardless of whether the vehicle charges or not.

Cooperative EV owners It is like the former case, except that EV owners allow up to 1 floating disconnection in the daytime parking interval to give the possibility of using that charger to other vehicles as opposed to the possibility of disconnecting only at the end of the morning period. It is worth noting that, in the night-time parking stay, this flexibility is not implemented as it is not considered practical for the EV owners to unplug vehicles during night time.

F. Nodal injections and grid model

A nodal injection consists of the total charging demand of EVs at that node, conventional demand connected at that node, and local PV generation. It includes both active and reactive parts. Nodal injections and their components are:

$$P_{nt}^{\text{node}} = P_{nt}^{\text{demand}} - P_{nt}^{\text{PV}} + P_{nt}^{\text{EV}} \tag{10a}$$
$$Q_{nt}^{\text{node}} = Q_{nt}^{\text{demand}} + Q_{nt}^{\text{EV}}. \tag{10b}$$

where $P_{nt}^{\text{demand}}, Q_{nt}^{\text{demand}}$ are the active and reactive power of the total demand, P_{nt}^{PV} is the PV generation (taken with the negative sign because it is generation), and $P_{nt}^{\text{EV}}, Q_{nt}^{\text{EV}}$ are the active and reactive power of the total EV charging demand at node n and time interval t. PV plants are assumed operated at a unitary power factor, so the reactive power contribution of PV plants does not appear in (10b).

Quantities P_{nt}^{demand}, Q_{nt}^{demand}, and P_{nt}^{PV} are input information, whereas P_{nt}^{EV} and Q_{nt}^{EV} depend on the EV charging patterns and are calculated as a function of the variables of the optimization problem. In particular, P_{nt}^{EV} is computed as:

$$P_{nt}^{\text{EV}} = \sum_{v=1}^{V} p_{nvt} \cdot p_{vt}^{\text{EV+}} \qquad \text{for all } t \text{ and } n. \tag{11}$$

The reactive power associated to the charging power (2) and (3) is

$$q_{vt}^{\text{EV+}} = p_{vt}^{\text{EV+}} \cdot \tan\phi, \qquad \forall t \text{ and } v \tag{12}$$

and the total reactive power at node n associated to the recharging demand is

$$Q_{nt}^{\text{EV}} = \sum_{v=1}^{V} p_{nvt} \cdot q_{vt}^{\text{EV+}} \qquad \text{for all } t \text{ and } n. \tag{13}$$

Because the grid voltage is constrained in a small range to respect statutory voltage limits, we use the approximation that all power injections are voltage independent ([1]).

Once nodal injections are known, nodal voltage magnitudes v_{tn}, current magnitudes i_{tl} in the lines with index $l = 1, \dots, L$, and power flow at the slack bus S_{t0} (assumed in this case as the grid connection point) can be calculated with load flow models as a function of the grid topology and cable characteristic. Because load flows are nonlinear and result in nonconvexities when included in an optimization problem, we use a linearized model based on sensitivity coefficients. Linearized grid models are indicated with the following notation:

$$v_{tn} = f_n\left(P_{t1}^{\text{node}}, \dots, P_{tN}^{\text{node}}, Q_{t1}^{\text{node}}, \dots, Q_{tN}^{\text{node}}\right) \tag{14a}$$
$$i_{tl} = g_l\left(P_{t1}^{\text{node}}, \dots, P_{tN}^{\text{node}}, Q_{t1}^{\text{node}}, \dots, Q_{tN}^{\text{node}}\right) \tag{14b}$$
$$S_{t0} = h_l\left(P_{t1}^{\text{node}}, \dots, P_{tN}^{\text{node}}, Q_{t1}^{\text{node}}, \dots, Q_{tN}^{\text{node}}\right) \tag{14c}$$

where f_n, g_l, and h_l are linear functions and highlight the dependency between grid quantities and nodal injections. The distribution grid's operational constraints can be expressed as (for all relevant indexes):

$$\underline{v} \leq v_{tn} \leq \overline{v} \tag{14d}$$
$$|i_{tl}| \leq \bar{i}_l \tag{14e}$$
$$S_{t0} \leq \overline{S}_0, \tag{14f}$$

[1] Voltage-dependent power injections could be accounted for by upgrading the linear grid model, as proposed in [20].

where $(\underline{v}, \overline{v})$ are statutory voltage levels, $\bar{i}_l$ is cable l's ampacity, and $\overline{S}_0$ is the kVA rating of the substation transformer.

In addition to the constraints in (14), in order to capture power limitations of the downstream electrical equipment (e.g., LV/MV transformers), we require nodal injections (10) to be smaller than kVA limit of node $\bar{S}_n$:

$$\left(P_{nt}^{\text{node}}\right)^2 + \left(Q_{nt}^{\text{node}}\right)^2 \leq (\bar{S}_n)^2. \tag{15}$$

Constraint (15) is nonlinear. By assuming a worst-case scenario of reactive power injections, we approximate (15) with the following linear expression:

$$-\bar{S}_n \cdot \underline{\cos}\phi_n \leq P_{nt}^{\text{node}} \leq \bar{S}_n \cdot \underline{\cos}\phi_n. \tag{16}$$

where $\underline{\cos}\phi$ is the lowest power factor of the nodal injection at node n.

G. Joint charging infrastructure planning and PV self consumption maximization

In this paper, we are interested in jointly optimizing the EV charging infrastructure in terms of required capital investments and PV self-consumption by shifting the recharging process of EVs when PV generation is available.

PV self-consumption of EVs at the power grid nodes can be promoted by incentivizing EVs to consume more power during time intervals when there is local PV production and vice-versa. This is modeled by minimizing the following objective function:

$$J^{\text{PV}} = \sum_{n=1}^{N} \sum_{t=1}^{T} \frac{1}{P_{nt}^{\text{PV}} + \epsilon} P_{nt}^{\text{EV}}, \tag{17}$$

where P_{nt}^{PV} is PV generation (an input of the problem), ϵ is a small coefficient to avoid dividing by zero when there is no PV generation, and P_{nt}^{EV} are the EV nodal injections, as defined in (11), which are the variables in this expression. In (17), when PV generation is available, recharging EVs will not impact much on the cost function value, whereas without PV generation, the cost function value will increase, thus penalizing this action and ultimately fostering PV self-consumption.

Cost functions (9) and (17) can be combined into a single one as:

$$J^{\text{total}} = J^{\text{chargers}} + k \cdot J^{\text{PV}} \tag{18}$$

where k is an input non-negative coefficient. The two components of the cost function above are different in nature, as the first is an economical cost and the second an incentive with no specific economic meaning. The weight k determines the importance of the terms. We note that when $k = 0$, we purely seek to minimize the cost of the infrastructure, whereas when k is large, the infrastructure is planned considering PV self-consumption mostly.

Ideally, we are interested in a problem solution that is stable for different values of k as this would indicate that the same charging infrastructure can accomplish both infrastructure cost minimization and PV self-consumption. In the results, we will specifically discuss under which conditions this happens.

H. Joint minimization of investment and operational costs with time-of-use electricity tariffs

Capital investment and operational costs can be jointly optimized by minimizing the following cost function:

$$J^{\text{total}} = J^{\text{chargers}} + \alpha \cdot \sum_{n=1}^{N} \sum_{t=1}^{T} P_{nt}^{\text{EV}} \cdot c_t \tag{19}$$

where c_t is a time-of-use electricity tariff at time t (assumed the same across the all grid) and the coefficient α ([2])

$$\alpha = \frac{\text{Service-life of chargers}}{\text{Optimization horizon}} \tag{20}$$

is a scale factor to make the two costs comparable.

I. Formulation of the optimization problems

The variables of the problem are the plugged-in and charging binary variables introduced in II-C. They are collected in the following vector $\boldsymbol{x}$:

$$\boldsymbol{x} = [s_{11}^{\text{charge}}, \ldots, s_{VT}^{\text{charge}} s_{11}^{\text{plugged}}, \ldots, s_{VT}^{\text{plugged}}] \in \mathbb{Z}_2^{V \times T}, \tag{21}$$

where $\mathbb{Z}_2 = \{0, 1\}$ is the set of binary number. As explained in the former sections, these variables are used to model the charging process of the vehicles, the number of required chargers, the nodal injections, EV owners' flexibility, and finally the grid constraints.

In addition to these variables, the initial state-of-charge values in (1b) are also problem decision variables. They are collected in the vector:

$$\boldsymbol{z} = [\text{SOC}_1(0), \ldots, \text{SOC}_V(0)] \in \mathbb{R}^V. \tag{22}$$

This is done with the objective of avoiding a problem solution that is sensitive to input information. For this set of variables, we also require the final SOC value to be larger than the initial one to avoid benefiting from the initial energy stock:

$$\text{SOC}_v(T) \geq \text{SOC}_v(0), \qquad \text{for all } v. \tag{23}$$

The problem formulation for the PV self-consumption consists of minimizing the cost function in (18) over the decision variables $\boldsymbol{x}$ and $\boldsymbol{z}$:

$$\min_{\boldsymbol{x} \in \mathbb{Z}_2^{V \times T}, \boldsymbol{z} \in \mathbb{R}^V} \left\{ J^{\text{chargers}} + k \cdot J^{\text{PV}} \right\} \tag{24a}$$

subject to the following constraints

Plugged-in only if parked (6)	$\forall t$ and v	(24b)
Charge only if plugged-in (4)	$\forall t$ and v	(24c)
SOC model (1) and charging power (2)	$\forall t$ and v	(24d)
Number of chargers model (8)	$\forall n$	(24e)
Forgetful (27) or cooperative owners (28)	$\forall v$	(24f)
Nodal injections (10)-(13)	$\forall t$ and n	(24g)
Grid constraints (14) and (16)	$\forall t$,n and l	(24h)

For the joint optimization of the capital and operational costs, the problem is the same as above except for the cost function that is replaced with (19).

[2]Discount rate is here neglected because it is not of primary interest in the results comparison.

III. Case study

A. Power distribution grid

The grid is the CIGRE benchmark grid for MV systems [21]. It is a three-phase 14-bus system ($N = 14$) with a nominal voltage of 20 kV and connected with the upper-grid level with two transformers, each serving a radial feeder, for a total rating of 50 kVA. The system is modeled with a single-phase equivalent model under the assumptions of balanced loads and transposed cables.

The grid topology of the system is shown in Fig. 1. The colored areas represent the parking locations of the vehicles. In particular, the nodes in Cluster 1 (purple area) are where the EVs are parked overnight, and those in Cluster 2 (green area) are where EVs are parked during the daytime. The time-of-use retail electricity tariff is approximated with the day-ahead electricity prices obtained from [22] and shown in Fig. 2 for 24 hours. This 1-day-long profile is replicated 5 times to obtain the profile of the 5-day-long optimization horizon adopted in the optimization.

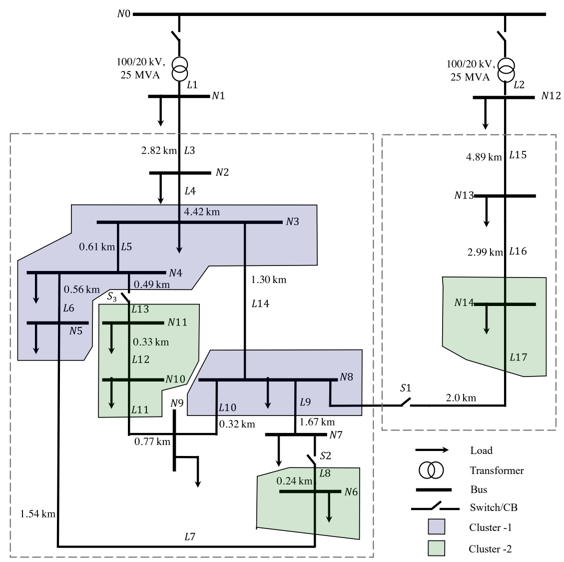

Fig. 1. Topology of the CIGRE European MV distribution network benchmark for residential system [21].

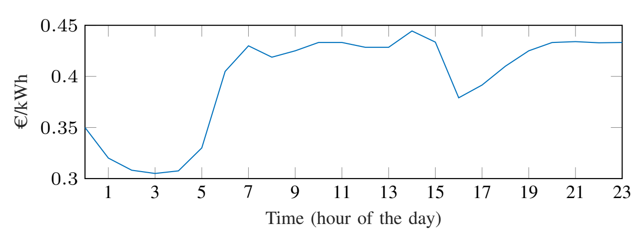

Fig. 2. Cost of the electricity in the month of July in France.

B. Nodal demand and PV generation

The nodal demand, P_{nt}^{demand} and Q_{nt}^{demand} in (10a), is simulated considering the 1-day long load coincidence factor described in [21], rescaled for the kVA nominal power of the nodes and then split into the active and reactive components with the nominal power factor of the nodes. Both nodal power factors and nominal powers are reported in Table I.

Table I shows the PV generation capacity installed at the various nodes of the grid. A total of 400 kWp of PV generation is installed in the network and connected to nodes 6, 10, and 11, corresponding to nodes where EVs are parked during the daytime. PV generation is simulated with first-principles models starting from irradiance time series as described in [23], considering clear-sky conditions and PV panels with tilt and azimuth optimized to guarantee the largest yield over the year.

TABLE I
NODAL NOMINAL DEMAND AND POWER FACTORS

Node	Apparent Power [kVA]	Power factor	Cluster	PV Gen [kW]
1	15’300	0.98	-	0
3	285	0.97	1	0
4	445	0.97	1	0
5	750	0.97	1	0
6	565	0.97	2	150
8	605	0.97	1	0
10	490	0.97	2	200
11	340	0.97	2	50
12	15’300	0.98	-	0
14	215	0.97	2	0

C. Length of the optimization horizon

The length of the optimization horizon, T, is 5 days. The time resolution of the time series is 1 hour. The 5-day-long input time series are obtained by replicating 5 times the 1-day-long nodal demand and PV generation trajectories. Despite the short planning horizon and the single scenario of PV generation and demand, this configuration already allows capturing the sensitivity of the charging infrastructure requirements with respect to PV self-consumption, as discussed later, which is the objective of this paper. More robust charging infrastructure requirements can be obtained with extended planning horizons and multiple scenarios of demand and PV generation, and it will be considered in future works. It is worth highlighting that the number of decision variables of the problem is proportional to T; because MILP problems are generally NP-hard, suitable strategies to limit the number of samples, such as scenario reduction techniques, should be identified to attain feasible computational times.

D. Number of vehicles and charger properties

A population of 800 EVs with 60 kWh batteries is considered ($V = 800$). The daily recharging demand of the EVs population is 17.1 ± 4.0 kWh (mean values and standard deviation), estimated using data from [24]. The discharging power $p_{vt}^{\text{EV-}}$, used to model the state-of-charge evolution in (1b), is positive and constant when the vehicle drives and zero when the EV is parked; in addition, it is such that its corresponding total energy amounts to the daily recharging demand described above (at the net of the charger efficiency). Level-1 chargers with a rating of 2.4 kVA are considered ($\bar{S} = 2.1$ kVA). The power factor of these chargers is 0.95 ($\cos\phi = 0.95$), their efficiency is 0.9 ($\eta = 0.9$), and unitary cost is ($\gamma = 11$). Faster (and more expensive) chargers with 20 kVA rating were also tested, but they were not conducive to improving performance (i.e., reducing capital investments or improving PV self-consumption). The service life of the charger is assumed 15 years.

E. Duration of the daytime intervals

The duration of the daytime parking intervals of the EVs could impact the results of the planning problem and PV self-consumption. In particular, longer daytime parking hours would allow charging more EVs in the central part of the day, coinciding with PV generation. In order to evaluate the sensitivity of the results to the duration of the daytime parking intervals, two scenarios are considered:

- base case parking stay: EVs are parked between 9 AM and 4 PM (plus/minus 0.5 hours).
- extended parking stay: EVs are parked between 5 AM and 8 PM (plus/minus 0.5 hours).

IV. RESULTS AND DISCUSSIONS

This section illustrates and compares the planning results obtained by optimizing for PV self-consumption and for joint capital and operational costs optimization. The optimization problems were solved with a MIP gap of 5% to decrease the computation time. Under this setting, both the problems took nearly one hour to solve in a computer with an Intel Xeon processor.

A. Optimizing for PV self-consumption

We first illustrate the impact of different values of k in the cost function; as a reminder from the previous sections, the coefficient k in (18) trades charging infrastructure costs for PV self-consumption.

Fig. 3 shows the values of the two components of the planning problem’s cost function in (18) for different values of k, forgetful/cooperative EV owners, and base case/extended parking intervals. The two cost components on the plot axis are J^{chargers} (capital investments required for the resulting charging infrastructure) and J^{PV} (achieved PV self-consumption, where lower values denoted improved PV self-consumption, and vice versa). Subfigures are now discussed in detail.

Fig. 3(a) shows that higher values of k attains lower values of J^{PV} (thus improving self consumption) but higher infrastructure costs J^{chargers}. This trend, found also in the remaining plots of Fig. 3, is to be expected because larger values of k in the cost function (18) gives more weight to PV self-consumption, and less to decreasing infrastructure costs.

Fig. 3(b) shows the evolution of the costs when introducing flexible drivers. Compared to Fig. 3(a), it can be seen that capital investment are marginally decreased, especially for $k \geq 0$.

Fig. 3(c) shows the evolution of the costs with extended duration of the daytime parking intervals for forgetful EV

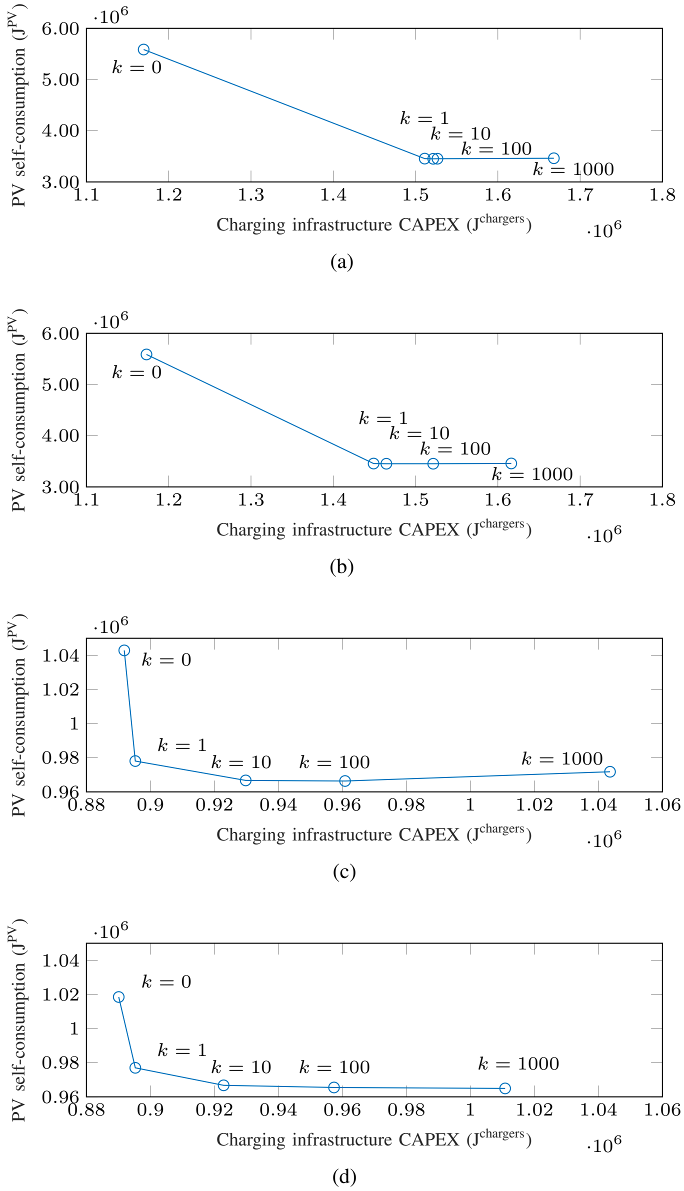

Fig. 3. Change in PV self-consumption J^{PV} (lower values denote better PV self-consumption) with the cost of the recharging infrastructure J^{chargers}, for increasing values of k in different scenarios: forgetful EV owners (a), Cooperative EV owners (b), forgetful EV owners with extended daytime parking intervals (c), and cooperative EV owners with extended daytime parking intervals (d).

owners. By comparing this figure against Fig. 3(a), it can be seen that:

- extending the daytime parking intervals leads to better PV self-consumption J^{PV}, as visible for $k = 0$.
- the value of the costs components in Fig. 3(c) is not as sensitive to variations of k as in Fig. 3(a).

Finally, Fig. 3(d) shows the evolution of the costs with extended duration of the daytime parking intervals and *flexible* EV owners. Compared to Fig. 3(b), it can be seen that increased flexibility of the EV owners leads to a (marginal) improvement of both the self-consumption and the infrastructure cost.

Tables II and III report the total number of installed chargers with the base case daytime parking intervals, forgetful EV owners, and increasing values of k when the recharging power of the chargers is on-off and modulated (equations (2) and (3)), respectively. It can be seen that the number of chargers generally increases with larger values of k, inline with the former discussion.

More interestingly, the two tables denote that the distribution of the chargers among clusters 1 and 2 changes between $k = 0$ and $k \geq 0$. In particular, with $k = 0$, chargers are mostly installed in Cluster 1 (where EVs are parked overnight), whereas with $k \geq 0$, chargers are mostly installed in Cluster 2 (where EVs are parked during the daytime). This denotes that promoting PV self-consumption from EVs requires developing a more pervasive charging infrastructure in those nodes where EVs are parked during the daytime.

Finally, it is also important to highlight that tables II and III feature similar values and trends, denoting that on-off or continuous modulation does not make a significant difference. This can be explained by the fact that at the aggregated level, modulating on-off a large number of vehicles will still achieve efficient congestion management and that continuous modulation might not be required.

TABLE II
TOTAL NUMBER OF CHARGERS AND DISTRIBUTION AMONG CLUSTERS FOR DIFFERENT VALUES OF k, BASE CASE DAYTIME PARKING INTERVALS, AND FORGETFUL EV OWNERS.

Node	k = 0	k = 1	k = 10	k = 100	k = 1000
3	49	58	62	57	60
4	95	88	79	92	119
5	169	115	116	108	148
6	101	215	219	219	224
8	118	51	58	60	63
10	52	205	205	205	205
11	73	117	119	117	125
14	21	27	24	27	23
Total	678	876	882	885	967
Cluster 1	64%	36%	36%	36%	40%
Cluster 2	36%	64%	64%	64%	60%

TABLE III
TOTAL NUMBER OF CHARGERS AND DISTRIBUTION AMONG CLUSTERS FOR DIFFERENT VALUES OF k, BASE CASE DAYTIME PARKING INTERVALS, AND FORGETFUL EV OWNERS FOR MODULATED CHARGING POWER.

Node	k = 0	k = 1	k = 10	k = 100	k = 1000
3	50	50	48	50	50
4	106	70	69	69	76
5	168	88	96	89	92
6	101	218	225	225	222
8	116	31	29	41	46
10	50	205	205	205	205
11	73	119	121	119	119
14	19	20	20	20	21
Total	683	801	813	818	831
Cluster 1	64%	30%	30%	30%	32%
Cluster 2	36%	70%	70%	70%	68%

Tables IV and V show the total number of chargers and distribution between clusters 1 and 2 under the condition when the parking intervals are extended for both forgetful and cooperative EV owners, respectively, for increasing values of k. It can be seen that, in both these cases, the charging infrastructure is nearly entirely developed in Cluster 2, where

EVs are parked for a longer duration during the daytime and where PV is available. Compared to the cases in Table II and III, it can also be seen that, for increasing values of k, first, it does not significantly impact the distribution of the chargers among the clusters, and second, it does not significantly impact the total number of required chargers to be installed. The fact that the properties of the charging infrastructure are similar for different values of k denotes that an EV charging infrastructure that is optimized for minimizing the investment cost is also capable of delivering "good performance" in terms of PV self-consumption.

TABLE IV
TOTAL NUMBER OF CHARGERS AND DISTRIBUTION AMONG CLUSTERS FOR DIFFERENT VALUES OF k, EXTENDED PARKING INTERVALS, AND FORGETFUL EV OWNERS.

Node	k = 0	k = 1	k = 10	k = 100	k = 1000
3	1	0	0	0	0
4	23	17	18	19	18
5	2	1	0	2	0
6	198	199	210	215	218
8	2	1	1	0	0
10	120	120	125	138	169
11	117	129	129	129	129
14	53	52	52	52	52
Total	516	519	535	555	586
Cluster 1	6%	4%	4%	4%	4%
Cluster 2	94%	96%	96%	96%	96%

TABLE V
TOTAL NUMBER OF CHARGERS AND DISTRIBUTION AMONG CLUSTERS FOR DIFFERENT VALUES OF k, EXTENDED PARKING INTERVALS, AND COOPERATIVE EV OWNERS.

Node	k = 0	k = 1	k = 10	k = 100	k = 1000
3	1	0	0	1	0
4	28	17	17	18	17
5	2	1	1	2	0
6	200	197	210	215	218
8	1	1	2	1	8
10	120	122	127	138	178
11	111	129	130	130	131
14	54	52	52	52	53
Total	517	519	539	557	605
Cluster 1	5%	4%	4%	4%	3%
Cluster 2	95%	96%	96%	96%	97%

B. Joint optimization of capital and operational costs

Table VI reports the number of chargers and their distributions for both normal and extended parking intervals and cooperative and forgetful EV owners. The following observations can be deduced:

- For the normal parking interval, chargers are predominantly present in Cluster 1, similarly to the case $k = 0$ in the former tables. However, the number of chargers, in this case, is much higher than in former tables (about twice as much). This can be explained by the fact that the planning problem finds it convenient to install more chargers in Cluster 1 (where EVs are parked overnight) in order to access lower electricity costs.
- For extended daytime parking intervals, where cars are parked longer in Cluster 2 and less in 1, chargers tend to be installed more in Cluster 2 than in Cluster 1. This is to be expected because installing chargers in Cluster 2 will enable access to lower electricity prices.

C. Comparison among all cases

Fig. 4 compares the percentage of chargers installed in Cluster 1 versus the total number of chargers for all the considered cases (for PV self-consumption, only the case for $k = 100$ is considered). It can be seen that the difference between cooperative and forgetful drivers is negligible when the other features are the same. Thus, this factor does not affect the charging infrastructure requirements in this case study.

It can be observed that the case with extended parking intervals and PV self-consumption requires the smallest charging infrastructure, mostly developed away from Cluster 1 (i.e., in Cluster 2). The remaining cases feature a charging infrastructure that is more similar with respect to each other, so we can conclude that it is more robust against possible changes of the planning objective as it features a more similar distribution and a total number of chargers.

TABLE VI
DISTRIBUTION OF NUMBER OF CHARGERS FOR DIFFERENT PARKING INTERVALS (800 EVS, 60 KWH BATTERY, 5 DAYS HORIZON, WITH SERVICE LIFE FACTOR).

Node	Normal interval		Extended interval	
	Forgetful chargers	Cooperative chargers	Forgetful chargers	Cooperative chargers
3	100	100	47	47
4	168	168	109	109
5	243	243	185	185
6	215	215	215	215
8	196	196	143	143
10	205	205	196	196
11	103	103	115	115
14	48	48	62	62
Total	1278	1278	1072	1072
Cluster 1	55%	55%	45%	45%
Cluster 2	45%	45%	55%	55%

V. CONCLUSIONS

This paper presented a MILP problem to site and size the EV charging infrastructure in a distribution grid to jointly minimize the required capital investments and maximize self-consumption of local PV generation. The formulation accounted for the operational constraints of the distribution grid (nodal voltages, line currents, and transformers' ratings), the EVs' recharging demand, and the flexibility of the EV owners in plugging and unplugging their vehicles.

A proof-of-concept by simulations was developed by considering the CIGRE benchmark system for MV grids, and a sensitivity analysis was performed to verify the impact of various factors on the planning results. Results showed that with short daytime parking intervals, improving PV self-consumption required installing more chargers (in addition to those required for overnight charging), resulting in higher infrastructure costs. However, when extending the daytime parking hours, it was found that a charging infrastructure developed in the nodes where EVs are parked during the

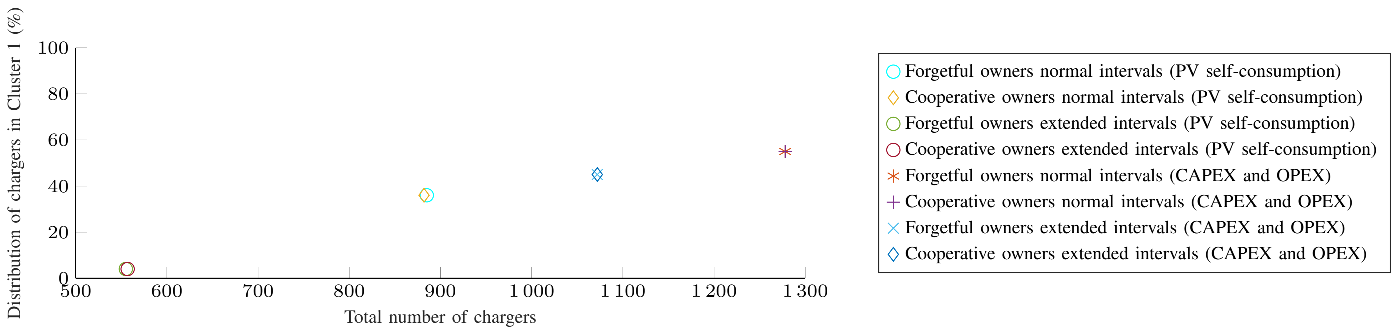

Fig. 4. Distribution and number of chargers in Cluster 1 for two optimization problems. Data obtained for the optimization with PV self-consumption are plotted for $k = 100$.

daytime is sufficient to meet the total recharging demand, ultimately leading to reduced EV charging infrastructure costs and improved PV self-consumption. The intermittent PV generation thus may provide a different configuration of charger distribution but certainly it will impact the planning results. Also, it is observed that when the objective is to minimize the nodal EV charging prices, it provides different results and incurs higher costs for the installation of the chargers. As a future work direction, one could embed all the sources of uncertainty in a single optimization problem with the objective of producing a charging infrastructure robust against uncertainty.

Appendix A
Modeling EV owners' flexibility

EV owners' availability and flexibility in plugging and unplugging their EVs into and from a charger, respectively, are modeled by adding additional constraints on variables s_{vt}^{plugged}. We start by modeling plugging and unplugging events into and from a chargers, which are modeled by detecting rising and falling edges of the variables s_{vt}^{plugged} as:

$$c_{vt}^{s} = \max\left(s_{vt}^{\text{plugged}} - s_{v(t-1)}^{\text{plugged}}, 0\right) \qquad \forall t \text{ and } v \tag{25}$$

$$d_{vt}^{s} = \max\left(s_{v(t-1)}^{\text{plugged}} - s_{vt}^{\text{plugged}}, 0\right) \qquad \forall t \text{ and } v. \tag{26}$$

We model two scenarios of EV owners' flexibility: *forgetful EV owners* and *cooperative EV owners*. To explain these scenarios, we specifically refer to the case study considered in this paper, which refers to a home-to-work-to-home commute, where EV owners drive their vehicles from an origin node to a destination node in the morning, and then back to the origin node in the evening. For vehicle v, let the time interval $(\tau_v^{(2)}, \tau_v^{(3)})$ encompass the duration of the morning commute and $(\tau_v^{(4)}, \tau_v^{(1)})$ the duration of the evening commute for vehicle v. The scenarios are as follows.

Forgetful EV owners In both the morning and afternoon commutes, owners plug their vehicles at the arrival time and unplug them at the departing time. In these intervals, the chargers are busy for the whole duration of the parking stay, regardless of whether the vehicle charges or not. This scenario is enforced by allowing unplugging events at the vehicles' departing times only (i.e., $\tau_v^{(2)}$ and $\tau_v^{(4)}$), and plugging events at the vehicles' arrival time only (i.e., $\tau_v^{(3)}$ and $\tau_v^{(1)}$). Formally, this reads as the following set of constraints:

$$d_{vt}^{s} \leq 0 \qquad \text{for all } t \text{ except } t = \tau_v^{(2)} \text{ and } t = \tau_v^{(4)} \tag{27a}$$

$$c_{vt}^{s} \leq 0 \qquad \text{for all } t \text{ except } t = \tau_v^{(3)} \text{ and } t = \tau_v^{(1)}. \tag{27b}$$

In words, connection and disconnection variables can become active only in the four indicated time intervals; outside these time intervals, binary connection and disconnection variables are forced to zero, allowing no EVs to plug or unplug.

Cooperative EV owners It is like the former scenario, except that EV owners allow up to 1 floating disconnection in the daytime parking interval (i.e., between $\tau_v^{(3)}$ and $\tau_v^{(4)}$) to give the possibility of using that charger to other vehicles as opposed to the possibility of disconnecting only at $\tau_v^{(4)}$. It is worth noting that, in the night-time parking stay, this flexibility is not implemented as it is not considered practical for the EV owners to unplug vehicles in the middle of the night. Connection and disconnection constraints read as:

$$c_{vt}^{s} \leq 0 \qquad \text{for all } t \text{ except } t = \tau_v^{(1)} \tag{28a}$$

$$d_{vt}^{s} \leq 0 \qquad \text{for all } t \text{ except } t = \tau_v^{(2)} \tag{28b}$$

$$\sum_{t=\tau_3}^{\tau_4} d_{vt}^{s} \leq 1. \tag{28c}$$

References

[1] M.-O. Metais, O. Jouini, Y. Perez, J. Berrada, and E. Suomalainen, "Too much or not enough? Planning electric vehicle charging infrastructure: a review of modeling options." Feb. 2021, working paper or preprint. [Online]. Available: https://hal.archives-ouvertes.fr/hal-03127266

[2] "Ministère de l'environnement de l'énergie et de lamer - La loi de transition Énergétique pour la croissance verte," accessed: 2021-03. [Online]. Available: https://bit.ly/3aRJ1Lb

[3] "Estimating electric vehicle charging infrastructure costs across major U.S. metropolitan areas," accessed: 2021-03. [Online]. Available: https://theicct.org/publications/charging-cost-US

[4] J. Widén, E. Wäckelgård, and P. D. Lund, "Options for improving the load matching capability of distributed photovoltaics: Methodology and application to high-latitude data," *Solar Energy*, vol. 83, no. 11, pp. 1953–1966, 2009. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0038092X09001741

[5] R. Luthander, J. Widén, D. Nilsson, and J. Palm, "Photovoltaic self-consumption in buildings: A review," *Applied Energy*, vol. 142, pp. 80–94, 2015. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0306261914012859

[6] G. Liu, L. Kang, Z. Luan, J. Qiu, and F. Zheng, "Charging station and power network planning for integrated electric vehicles (EVs)," *Energies*, vol. 12, no. 13, 2019.

[7] Y. Xiang, J. Liu, R. Li, F. Li, C. Gu, and S. Tang, "Economic planning of electric vehicle charging stations considering traffic constraints and load profile templates," *Applied Energy*, vol. 178, pp. 647–659, 2016.

[8] J. Li, X. Sun, Q. Liu, W. Zheng, H. Liu, and J. A. Stankovic, "Planning electric vehicle charging stations based on user charging behavior," in *2018 IEEE/ACM Third International Conference on Internet-of-Things Design and Implementation (IoTDI)*, 2018, pp. 225–236.

[9] W. Gan, M. Shahidehpour, J. Guo, W. Yao, A. Paaso, L. Zhang, and J. Wen, "Two-stage planning of network-constrained hybrid energy supply stations for electric and natural gas vehicles," *IEEE Transactions on Smart Grid*, vol. 12, no. 3, pp. 2013–2026, 2021.

[10] W. Gan, M. Shahidehpour, M. Yan, J. Guo, W. Yao, A. Paaso, L. Zhang, and J. Wen, "Coordinated planning of transportation and electric power networks with the proliferation of electric vehicles," *IEEE Transactions on Smart Grid*, vol. 11, no. 5, pp. 4005–4016, 2020.

[11] A. Y. S. Lam, Y.-W. Leung, and X. Chu, "Electric vehicle charging station placement: Formulation, complexity, and solutions," *IEEE Transactions on Smart Grid*, vol. 5, no. 6, pp. 2846–2856, 2014.

[12] C. Luo, Y.-F. Huang, and V. Gupta, "Placement of EV charging stations—balancing benefits among multiple entities," *IEEE Transactions on Smart Grid*, vol. 8, no. 2, pp. 759–768, 2017.

[13] Z. Hu and Y. Song, "Distribution network expansion planning with optimal siting and sizing of electric vehicle charging stations," in *2012 47th International Universities Power Engineering Conference (UPEC)*, 2012, pp. 1–6.

[14] X. Yanhai, A. Bo, and K. Sarit, "Electric vehicle charging strategy study and the application on charging station placement," *Autonomous Agents and Multi-Agent Systems*, vol. 35, no. 3, 2021.

[15] J. Zhang, S. Wang, C. Zhang, F. Luo, Z. Y. Dong, and Y. Li, "Planning of electric vehicle charging stations and distribution system with highly renewable penetrations," *IET Electrical Systems in Transportation*, vol. 11, no. 3, pp. 256–268, 2021. [Online]. Available: https://ietresearch.onlinelibrary.wiley.com/doi/abs/10.1049/els2.12022

[16] H. Wang, Q. Huang, C. Zhang, and A. Xia, "A novel approach for the layout of electric vehicle charging station," in *The 2010 International Conference on Apperceiving Computing and Intelligence Analysis Proceeding*, 2010, pp. 64–70.

[17] D. Yan and C. Ma, "Stochastic planning of electric vehicle charging station integrated with photovoltaic and battery systems," *IET Generation, Transmission & Distribution*, vol. 14, no. 19, pp. 4217–4224, 2020. [Online]. Available: https://ietresearch.onlinelibrary.wiley.com/doi/abs/10.1049/iet-gtd.2019.1737

[18] B. Mukherjee and F. Sossan, "Optimal planning of single-port and multi-port charging stations for electric vehicles in medium voltage distribution networks," *arXiv preprint arXiv:2111.07100*, 2021.

[19] J. Van Roy, "Electric vehicle charging integration in buildings: Local charging coordination and dc grids," Ph.D. dissertation, KU Leuven, 2015.

[20] S. Fahmy and M. Paolone, "Analytical computation of power grids' sensitivity coefficients with voltage-dependent injections," in *2021 IEEE Madrid PowerTech*, 2021, pp. 1–6.

[21] CIGRE' Task Force C6.04.02, "Benchmark systems for network integration of renewable and distributed energy resources," CIGRE International Council on large electric systems, Tech. Rep., July 2009.

[22] "Da electricity prices in france - epexspot." accessed: 2021-07. [Online]. Available: https://www.epexspot.com/en/market-data

[23] F. Sossan, E. Scolari, R. Gupta, and M. Paolone, "Solar irradiance estimations for modeling the variability of photovoltaic generation and assessing violations of grid constraints: A comparison between satellite and pyranometers measurements with load flow simulations," *Journal of Renewable and Sustainable Energy*, vol. 11, no. 5, p. 056103, 2019.

[24] "Test-an-EV project: Electrical vehicle (EV) data," http://mclabprojects.di.uniroma1.it/smarthgnew/Test-an-EV/?EV-code=EV8, accessed: 2020-02.